VISCOUS LIQUID FLOW IN A NARROW GAP

BETWEEN TWO NONPLANAR SURFACES

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The effect of gap median surface form on the character of Hill–Show type flow ($Re \ll 1$) is demonstrated.

To describe flow in a gap between neighboring nonplanar surfaces we introduce a system of curvilinear orthogonal coordinates x^1, x^2, x^3, with the coordinate surface $x^3 = 0$ being the median surface of the gap. As is well known [1], the equations of continuity and steady state motion of a viscous, weightless, incompressible liquid in such curvilinear orthogonal coordinates have the following form:

$$\frac{\partial (H_{2*}H_{3*}v_{x^1})}{\partial x^1} + \frac{\partial (H_{3*}H_{1*}v_{x^2})}{\partial x^2} + \frac{\partial (H_{1*}H_{2*}v_{x^3})}{\partial x^3} = 0;$$

$$\rho \sum_{k=1}^{3} \left(\frac{v_{x^k}}{H_{k*}} \frac{\partial v_{x^i}}{\partial x^k} - \frac{v_{x^k}^2}{H_{i*}H_{k*}} \frac{\partial H_{k*}}{\partial x^i} + \frac{v_{x^k} v_{x^i}}{H_{k*}H_{i*}} \frac{\partial H_{i*}}{\partial x^k} \right) = \quad (1)$$

$$= -\frac{1}{H_{i*}} \frac{\partial p}{\partial x^i} + \frac{1}{H_{i*}} \sum_{k=1}^{3} \left[\frac{1}{H_{1*}H_{2*}H_{3*}} \frac{\partial}{\partial x^k} \left(\frac{H_{1*}H_{2*}H_{3*}}{H_{k*}} \tau_{ik} \right) - \tau_{kk} \frac{\partial \ln H_{k*}}{\partial x^i} \right];$$

$$\tau_{ik} = \rho\nu \left[\frac{1}{H_{k*}} \frac{\partial v_{x^i}}{\partial x^k} + \frac{1}{H_{i*}} \frac{\partial v_{x^k}}{\partial x^i} - \frac{1}{H_{i*}H_{k*}} \left(v_{x^i} \frac{\partial H_{i*}}{\partial x^k} + v_{x^k} \frac{\partial H_{k*}}{\partial x^i} \right) + 2\delta_k^i \sum_{\lambda=1}^{3} \frac{v_{x^\lambda}}{H_{\lambda*}} \frac{\partial \ln H_{i*}}{\partial x^\lambda} \right]. \quad (2)$$

We take the gap size constant and equal to 2h, and construct the coordinate system such that along the median surface ($x^3 = 0$) the coefficient $H_{3*} = 1$. To write Eqs. (1), (2) in dimensionless form we introduce new variables: $H_{j*}dx^j = r_0 H_j d\xi^j$; $dx^3 = h d\zeta$; $h/r_0 = \varepsilon$; $j = 1, 2$; $v_{x^j} = Vu_j$; $v_{x^3} = Vw$; $p = (\rho\nu r_0 V/h^2)\Pi$; $Re_* = Vh/\nu$.

We will assume that the linear scale r_0 characterizing the curvature of the median surface is also the characteristic scale of the flow on this surface, i.e., that for any hydrodynamic parameter A on the contour Γ which bounds the flow region under consideration the inequality $|A(\xi_1^1, \xi_1^2) - A(\xi_2^1, \xi_2^2)| \ll |A(\xi_1^1, \xi_1^2)|$ is satisfied, if $\delta \ll r_0$, where δ is the shortest distance between the points (ξ_1^1, ξ_1^2) and (ξ_2^1, ξ_2^2) along the median surface ($\zeta = 0$ along a geodesic line). From (2) we have

$$\tau_{j3} = \frac{\rho\nu V}{h} \left[\frac{\partial u_j}{\partial \zeta} + O\left(\varepsilon \frac{\partial w}{\partial \xi^j} \right) \right]; \quad \tau_{33} = \frac{\rho\nu V}{h} \left[2 \frac{\partial w}{\partial \xi} + O(\varepsilon) \right];$$

$$\frac{\partial \tau_{3k}}{\partial x^k} = \frac{\rho\nu V}{h} \left[\varepsilon \frac{\partial^2 u_k}{\partial \xi^k \partial \zeta} + O(\varepsilon^2) \right].$$

The gap 2h is assumed so small that the number $Re = \varepsilon Re_* \ll 1$, while the ratio of the characteristic dimensions $\varepsilon = \gamma Re^3 [\gamma = O(1)]$. Then Eqs. (1), (2) can be rewritten in the form

$$\frac{\partial w}{\partial \zeta} + \frac{w}{H_1 H_2} \frac{\partial (H_1 H_2)}{\partial \zeta} = -\frac{\gamma Re^3}{H_1 H_2} \left[\frac{\partial (H_2 u_1)}{\partial \xi^1} + \frac{\partial (H_1 u_2)}{\partial \xi^2} \right]; \quad (3)$$

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$$\frac{\partial^2 u_j}{\partial \zeta^2} = \frac{1}{H_1}\frac{\partial \Pi}{\partial \xi^j} + \mathrm{Re}\left[\sum_{k=1}^{2}\left(\frac{u_k}{H_k}\frac{\partial u_k}{\partial \xi^k} - \frac{u_k^2}{H_j H_k}\frac{\partial H_k}{\partial \xi^j} + \frac{u_k u_j}{H_k H_j}\frac{\partial H_j}{\partial \xi^k}\right) + \frac{w u_j}{H_j}\frac{\partial H_j}{\gamma \partial \zeta}\right]; \quad (4)$$

$$\frac{\partial \Pi}{\partial \zeta} = O(\mathrm{Re}\, w^2) + O(\mathrm{Re}^3 w) + O(\mathrm{Re}^4 u_k^2) + O(\mathrm{Re}^6 u_k); \quad (5)$$

$$\frac{\partial H_j}{\varepsilon \partial \zeta} = \frac{\partial H_{j*}}{\partial x^3} = O(1); \quad j,\ k = 1,\ 2. \quad (6)$$

The velocity components u_j, w must satisfy the adhesion boundary conditions

$$u_j = 0,\ w = 0 \ \text{ at } \ \zeta = \pm 1. \quad (7)$$

We will seek a solution of the corresponding problem for the hydrodynamically stabilized flow region in the form of a regular expansion in the small parameter Re: $u_j = u_j^{(0)} + \mathrm{Re} u_j^{(1)} + \ldots$; $w = w^{(0)} + \mathrm{Re}\, w^{(1)} + \ldots$; $\Pi = \Pi^{(0)} + \mathrm{Re}\, \Pi^{(1)} + \ldots$.

From Eqs. (3), (5), (7):

$$w^{(0)} = w^{(1)} = w^{(2)} = 0; \ \frac{\partial \Pi^{(i)}}{\partial \zeta} = 0 \ \text{ at } \ i \leqslant 3. \quad (8)$$

Thus, as a zeroth iteration we obtain a natural generalization of the equation describing Hill–Show flow in slot channels with a nonplanar median surface:

$$\frac{\partial^2 u_j^{(0)}}{\partial \zeta^2} = \frac{1}{H_j}\frac{\partial \Pi^{(0)}}{\partial \xi^j},\ u_j^{(0)} = 0 \ \text{ at } \ \zeta = \pm 1. \quad (9)$$

It follows from Eq. (6) that the change in the coefficient H_j across the gap is of the order of magnitude of $\varepsilon = \gamma \mathrm{Re}^3$. Therefore in determining the first three iterations it can be assumed that $H_j = H_j(\xi^1, \xi^2, 0)$, while from Eqs. (8), (9) we have

$$u_j^{(0)} = -\frac{1-\zeta^2}{2}\frac{1}{H_j}\frac{\partial \Pi^{(0)}}{\partial \xi^j}. \quad (10)$$

Integrating Eq. (3) over ζ with consideration of the second boundary condition of Eq. (7), we obtain an equation for the specific liquid fluxes

$$\frac{\partial (H_2 q_1)}{\partial \xi^1} + \frac{\partial (H_1 q_2)}{\partial \xi^2} = O(\varepsilon) = O(\mathrm{Re}^3); \quad (11)$$

$$2q_j = \int_{-1}^{1} u_j(\xi^1,\ \xi^2,\ \zeta)\, d\zeta = 2(q_j^{(0)} + \mathrm{Re}\, q_j^{(1)} + \mathrm{Re}^2 q_j^{(2)} + \ldots). \quad (12)$$

But in light of Eq. (10)

$$q_j^{(0)} = -\frac{1}{3}\frac{1}{H_j}\frac{\partial \Pi^{(0)}}{\partial \xi^j} \quad (13)$$

and consequently,

$$\frac{\partial}{\partial \xi^1}\left(\frac{H_2}{H_1}\frac{\partial \Pi^{(0)}}{\partial \xi^1}\right) + \frac{\partial}{\partial \xi^2}\left(\frac{H_1}{H_2}\frac{\partial \Pi^{(0)}}{\partial \xi^2}\right) = 0. \quad (14)$$

If the pressure distribution is specified along the contour Γ bounding the flow region under consideration the function $\Pi^{(0)}(\xi^1, \xi^2)$ can be defined by solution of the Dirichlet problem for Eq. (14). With a known distribution of the normal component $q_n = qn$ along the contour Γ finding the pressure coefficient $\Pi^{(0)}$ in the region S of the median surface reduces to solution of the Neiman problem for Eq. (14)

$$\frac{1}{H_1}\frac{\partial \Pi^{(0)}}{\partial \xi^1}\cos(\bar{n},\ \bar{e}_1) + \frac{1}{H_2}\frac{\partial \Pi^{(0)}}{\partial \xi^2}\cos(\bar{n},\ e_2) = 3q_n \ \text{ on } \ \Gamma. \quad (15)$$

On the basis of Eqs. (10)-(12) we have

$$\frac{\partial (H_2 u_1^{(0)})}{\partial \xi^1} + \frac{\partial (H_1 u_2^{(0)})}{\partial \xi^2} = 0, \quad (16)$$

while it follows from Eqs. (3), (6), (7) that $w^{(3)} = 0$. Therefore the equation for the first approximation can be written in the form

$$\frac{\partial^2 u_j^{(1)}}{\partial \zeta^2} = \frac{1}{H_j}\left\{\frac{\partial \Pi^{(1)}}{\partial \xi^j} + H_j \sum_{k=1}^{2}\left[\frac{u_k^{(0)}}{H_k}\frac{\partial u_j^{(0)}}{\partial \xi^k} - \frac{(u_k^{(0)})^2}{H_j H_k}\frac{\partial H_k}{\partial \xi^j} + \frac{u_k^{(0)} u_j^{(0)}}{H_k H_j}\frac{\partial H_j}{\partial \xi^k}\right]\right\}. \tag{17}$$

The second term in curly brackets can be represented in tensor form: $(U^{(0)})^k (U^{(0)})_{j,k}$ [1]. Here $(U^{(0)})^i$, $(U^{(0)})_j$ are respectively the contravariant and covariant components of the vector, the physical components of which are equal to $u_j^{(0)}$; $u_j^{(0)} = H_j (U^{(0)})^j = H_j^{-1} (U^{(0)})_j$; $(U^{(0)})_{j,k}$ is the covariant derivative of the covariant vector; we perform a summation over the indices appearing twice in the tensor equations. Since

$$(U^{(0)})^k (U^{(0)})_{j,k} = [(U^{(0)})_{j,k} - (U^{(0)})_{k,j}](U^{(0)})^k + \frac{1}{2}\frac{\partial}{\partial \xi^j}[(U^{(0)})_k (U^{(0)})^k],$$

and it follows from Eq. (10) that $(U^{(0)})_{j,k} - (U^{(0)})_{k,j} = 0$,

$$(U^{(0)})_k (U^{(0)})^k = \frac{(1-\zeta^2)^2}{4}\left[\left(\frac{1}{H_1}\frac{\partial \Pi^{(0)}}{\partial \xi^1}\right)^2 + \left(\frac{1}{H_2}\frac{\partial \Pi^{(0)}}{\partial \xi^2}\right)^2\right].$$

Consequently, Eq. (17) can be rewritten in the form

$$\frac{\partial^2 u_j^{(1)}}{\partial \zeta^2} = \frac{1}{H_j}\frac{\partial \Pi^{(1)}}{\partial \xi^j} + \frac{(1-\zeta^2)^2}{8}\frac{1}{H_j}\frac{\partial}{\partial \xi^j}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2$$

and with consideration of the first boundary condition of Eq. (7) we obtain

$$u_j^{(1)} = -\frac{1-\zeta^2}{2}\frac{1}{H_j}\frac{\partial \Pi^{(1)}}{\partial \xi^j} - \frac{1}{16}\left(\frac{1-\zeta^6}{15} - \frac{1-\zeta^4}{3} + 1 - \zeta^2\right)\frac{1}{H_j}\frac{\partial}{\partial \xi^j}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2; \quad q_j^{(1)} = \frac{1}{H_j}\frac{\partial \Phi^{(1)}}{\partial \xi^j}; \tag{18}$$

$$\Phi^{(1)} = -\frac{\Pi^{(1)}}{3} - \frac{1}{35}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2.$$

In view of continuity equation (11)

$$\frac{\partial}{\partial \xi^1}\left(\frac{H_2}{H_1}\frac{\partial \Phi^{(1)}}{\partial \xi^1}\right) + \frac{\partial}{\partial \xi^2}\left(\frac{H_1}{H_2}\frac{\partial \Phi^{(1)}}{\partial \xi^2}\right) = 0. \tag{19}$$

If the distribution of the normal component of the specific liquid fluxes q_n is specified along the contour Γ, then

$$\frac{1}{H_1}\frac{\partial \Phi^{(1)}}{\partial \xi^1}\cos(\bar{n}, \bar{e}_1) + \frac{1}{H_2}\frac{\partial \Phi^{(1)}}{\partial \xi^2}\cos(\bar{n}, \bar{e}_2) = 0 \text{ on } \Gamma$$

and in view of the uniqueness of the solution of the internal Neiman problem for Eq. (19) the function $\Phi^{(1)}(\xi^1, \xi^2) = 0$ in the entire region S (limited by the contour Γ) and the pressure coefficient $\Pi^{(1)}(\xi^1, \xi^2)$ can be calculated with the expression

$$\Pi^{(1)} = -\frac{3}{35}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2. \tag{20}$$

Given the pressure distribution on the contour Γ the potential $\Phi^{(1)}(\xi^1, \xi^2)$ can be defined by solution of the Dirichlet problem for Eq. (19) with the boundary condition

$$\Phi^{(1)} = -\frac{1}{35}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2 \text{ on } \Gamma. \tag{21}$$

The following iteration has the form

$$\frac{\partial^2 u_j^{(2)}}{\partial \zeta^2} = \frac{1}{H_j}\left[\frac{\partial \Pi^{(2)}}{\partial \xi^j} + H_j \sum_{k=1}^{2}\left(\frac{u_k^{(0)}}{H_k}\frac{\partial u_j^{(1)}}{\partial \xi^k} + \frac{u_k^{(1)}}{H_k}\frac{\partial u_j^{(0)}}{\partial \xi^k} - 2\frac{u_k^{(0)} u_k^{(1)}}{H_j H_k}\frac{\partial H_k}{\partial \xi^j} + \frac{u_k^{(0)} u_j^{(1)} + u_k^{(1)} u_j^{(0)}}{H_j H_k}\frac{\partial H_j}{\partial \xi^k}\right)\right] + \frac{w^{(4)}}{\gamma}\frac{\partial u_j^{(0)}}{\partial \zeta}. \tag{22}$$

Substituting Eqs. (18), (20) in the relationship obtained from continuity equation (3)

$$\frac{\partial w^{(4)}}{\partial \zeta} = -\frac{\gamma}{H_1 H_2}\left[\frac{\partial (H_2 u_1^{(1)})}{\partial \xi^1} + \frac{\partial (H_1 u_2^{(1)})}{\partial \xi^2}\right]$$

and using the second boundary condition of Eq. (7) we find the coefficient $w^{(4)}$ for a flux distribution q_n specified on the contour Γ

$$w^{(4)} = \frac{\gamma}{16}\left[\frac{1}{15}\left(\zeta - \frac{\zeta^7}{7}\right) - \frac{1}{3}\left(\zeta - \frac{\zeta^5}{5}\right) + \frac{11}{35}\left(\zeta - \frac{\zeta^3}{3}\right)\right] \times$$
$$\times \sum_{j=1}^{2} \frac{1}{H_1 H_2} \frac{\partial}{\partial \xi^j}\left[\frac{H_i}{H_j} \frac{\partial}{\partial \xi^j} \sum_{k=1}^{2}\left(\frac{1}{H_k} \frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right]; \quad i = 1, 2; \qquad i \neq j. \tag{23}$$

The second term in square brackets in Eq. (22) can be written in tensor form:

$$(U^{(0)})^k (U^{(1)})_{j,k} + (U^{(1)})^k (U^{(0)})_{j,k} = (U^{(0)})^k [(U^{(1)})_{j,k} - (U^{(1)})_{k,j}]$$
$$+ (U^{(1)})^k [(U^{(0)})_{j,k} - (U^{(0)})_{k,j}] + \frac{\partial}{\partial \xi^j} [(U^{(0)})_k (U^{(1)})^k].$$

It then follows from Eqs. (10), (18) that $(U^{(i)})_{j,k} - (U^{(i)})_{k,j} = 0$ at $i = 0, 1$, so that the corresponding term is equal to

$$\frac{\partial}{\partial \xi^j}(u_1^{(0)} u_1^{(1)} + u_2^{(0)} u_2^{(1)}). \tag{24}$$

Substituting Eqs. (23), (24) in Eq. (22) and integrating the equation obtained twice over ζ we then find, using the first boundary condition of Eq. (7),

$$u_j^{(2)} = -\frac{1-\zeta^2}{2} \frac{1}{H_j} \frac{\partial \Pi^{(2)}}{\partial \xi^j} + \frac{1}{32}\left[\frac{1}{15}(a_4 + a_8 - a_2 - a_{10}) - \right.$$
$$\left. - \frac{1}{3}(a_4 + a_6 - a_2 - a_{10}) - \frac{11}{35}(a_2 + a_6 - a_4)\right] \times$$
$$\times \frac{1}{H_j} \frac{\partial}{\partial \xi^j}\left[\sum_{i=1}^{2} \frac{1}{H_i} \frac{\partial \Pi^{(0)}}{\partial \xi^i} \frac{1}{H_i} \frac{\partial}{\partial \xi^i} \sum_{k=1}^{2}\left(\frac{1}{H_k} \frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right] - \frac{1}{16}\left[\frac{1}{15}\left(a_4 - \frac{a_{10}}{7}\right) - \frac{1}{3}\left(a_4 - \frac{a_8}{5}\right) + \right.$$
$$\left. + \frac{11}{35}\left(a_4 - \frac{a_5}{3}\right)\right] \frac{1}{H_j} \frac{\partial \Pi^{(0)}}{\partial \xi^j} \sum_{i=1}^{2} \frac{1}{H_1 H_2} \frac{\partial}{\partial \xi^i}\left[\frac{H_l}{H_i} \frac{\partial}{\partial \xi^i} \sum_{k=1}^{2}\left(\frac{1}{H_k} \frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right];$$

$$q_j^{(2)} = -\frac{1}{3} \frac{1}{H_j} \frac{\partial \Pi^{(2)}}{\partial \xi^j} - \frac{52}{363\,825}\left\{\frac{1}{H_j} \frac{\partial}{\partial \xi^j} \times \right.$$
$$\times \left[\sum_{i=1}^{2} \frac{1}{H_i} \frac{\partial \Pi^{(0)}}{\partial \xi^i} \frac{1}{H_i} \frac{\partial \Pi^{(0)}}{\partial \xi^i} \frac{1}{H_i} \frac{\partial}{\partial \xi^i} \sum_{k=1}^{2}\left(\frac{1}{H_k} \frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right] +$$
$$\left. + \frac{1}{2} \frac{1}{H_j} \frac{\partial \Pi^{(0)}}{\partial \xi^j} \sum_{i=1}^{2} \frac{1}{H_1 H_2} \frac{\partial}{\partial \xi^i}\left[\frac{H_l}{H_i} \frac{\partial}{d \xi^i} \sum_{k=1}^{2}\left(\frac{1}{H_k} \frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right]\right\};$$

$$a_n = \frac{1-\zeta^n}{n(n-1)}; \quad l = 1, 2; \quad l \neq i.$$

In light of continuity equation (11)

$$-\frac{1}{3} \sum_{j=1}^{2} \frac{1}{H_1 H_2} \frac{\partial}{\partial \xi^j}\left(\frac{H_m}{H_j} \frac{\partial \Pi^{(2)}}{\partial \xi^j}\right) - \frac{52}{363\,825}\left\{\sum_{j=1}^{2} \frac{1}{H_1 H_2} \frac{\partial}{\partial \xi^j}\left[\frac{H_m}{H_j} \frac{\partial}{\partial \xi^j}\left(\sum_{k=1}^{2} \frac{1}{H_k} \frac{\partial \Pi^{(0)}}{\partial \xi^k} \frac{1}{H_k} \frac{\partial}{\partial \xi^k} \times \right.\right.\right.$$

$$\times \sum_{i=1}^{2}\left(\frac{1}{H_i}\frac{\partial \Pi^{(0)}}{\partial \xi^i}\right)^2\Bigg)\Bigg] + \frac{1}{2}\sum_{j=1}^{2}\frac{1}{H_j}\frac{\partial \Pi^{(0)}}{\partial \xi^j}\frac{1}{H_j}\frac{\partial}{\partial \xi^j}\times$$

$$\times \sum_{i=1}^{2}\frac{1}{H_1H_2}\frac{\partial}{\partial \xi^i}\left[\frac{H_l}{H_i}\frac{\partial}{\partial \xi^i}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right]\Bigg\} = 0;\quad l = 1,\ 2;\quad l \neq i.$$

On the basis of Eq. (14) the last term in curly brackets can be represented by the sum:

$$\frac{1}{4}\sum_{j=1}^{2}\frac{1}{H_1H_2}\frac{\partial}{\partial \xi^j}\left\{\frac{H_m}{H_j}\frac{\partial}{\partial \xi^j}\left[\Pi^{(0)}\sum_{i=1}^{2}\frac{1}{H_1H_2}\frac{\partial}{\partial \xi^i}\left[\frac{H_l}{H_i}\frac{\partial}{\partial \xi^i}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right]\right]\right\}.$$

Consequently, the vector

$$q_j^{(2)} = \frac{1}{H_j}\frac{\partial \Phi^{(2)}}{\partial \xi^j} + q_{j2}.$$

The potential $\Phi^{(2)}$ like $\Phi^{(1)}$ satisfies Eq. (19), while the vector

$$q_{j2} = \frac{26}{363\,825}\left\{\frac{1}{2}\frac{1}{H_j}\frac{\partial}{\partial \xi^j}\left[\Pi^{(0)}\sum_{i=1}^{2}\frac{1}{H_1H_2}\frac{\partial}{\partial \xi^i}\times\right.\right.$$

$$\left.\left.\times\left(\frac{H_l}{H_i}\frac{\partial}{\partial \xi^i}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right)\right] - \frac{1}{H_j}\frac{\partial \Pi^{(0)}}{\partial \xi^j}\sum_{i=1}^{2}\frac{1}{H_1H_2}\frac{\partial}{\partial \xi^i}\left[\frac{H_l}{H_i}\frac{\partial}{\partial \xi^i}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right]\right\}.$$

If the liquid flux distribution q_n is specified along the contour Γ then determination of the function $\Phi^{(2)}(\xi^1, \xi^2)$ reduces to solution of the Neiman problem for Eq. (19). On the contour Γ the conditions

$$\frac{1}{H_1}\frac{\partial \Phi^{(2)}}{\partial \xi^1}\cos(\bar{n}, \bar{e}_1) + \frac{1}{H_2}\frac{\partial \Phi^{(2)}}{\partial \xi^2}\cos(\bar{n}, \bar{e}_2) + q_{12}\cos(\bar{n}, \bar{e}_1) + q_{22}\cos(\bar{n}, \bar{e}_2) = 0 \tag{25}$$

must be satisfied. The coefficient of the series expansion for the pressure

$$\Pi^{(2)} = -3\Phi^{(2)} - \frac{52}{363\,825}\left\{\sum_{i=1}^{2}\frac{1}{H_i}\frac{\partial \Pi^{(0)}}{\partial \xi^i}\frac{1}{H_i}\frac{\partial}{\partial \xi^i}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2 + \right.$$
$$\left. + \frac{1}{4}\Pi^{(0)}\sum_{i=1}^{2}\frac{1}{H_1H_2}\frac{\partial}{\partial \xi^i}\left[\frac{H_l}{H_i}\frac{\partial}{\partial \xi^i}\sum_{k=1}^{2}\left(\frac{1}{H_k}\frac{\partial \Pi^{(0)}}{\partial \xi^k}\right)^2\right]\right\}. \tag{26}$$

The three terms of the expansion $u_j^{(m)}$ found above are analogous in structure to the corresponding terms of the expansion describing flow in gaps with a planar median surface [2] and are a generalization of the latter to the case of an arbitrary surface where an orthogonal coordinate system has been introduced. The functions $\Phi^{(0)}$, $\Phi^{(1)}$, and $\Phi^{(2)}$ satisfy one and the same Eq. (14) (or Eq. (19)), which is a generalization of the two-dimensional Laplace equation to nonplanar surfaces. Therefore in solving the corresponding boundary problems in some cases it is possible to effectively utilize a conformal projection of the region S of the median plane onto a region of some other surface for which the corresponding boundary problem can be solved easily. Thus, for example, a stereographic projection [3] permits conformal mapping of a portion of a spherical surface limited by an arbitrary set of arcs of circles onto a planar region with a boundary consisting of arcs of circles and straight line segments. If the median surface is evolvable, then before solving the problem it is desirable to map the region S onto a plane which allows use of the methods of analytic function theory. In [4, 5] such an approach was used to analyze flows of vapor which develop with a double phase transition in gaps between cylindrical and conical shells in temperature stabilization of large size objects. In [6] analogous problems related to heating of thin-walled shells were solved by the virtual source (sink) method and expansion of the unknown function in eigenfunctions of specially constructed equations.

NOTATION

v_{x^i}, physical components of velocity vector (i = 1, 2, 3); x^1, x^2, x^3, orthogonal curvolinear coordinate system; $H_i{*}$, Lamé coefficients; $\delta_k{}^i$, Kronecker symbols; 2h, gap size; r_0, linear scale characterizing curvature of median surface; $\varepsilon = h/r_0$; $H_j{*}dx^j = r_0H_jd\xi^j$ (j = 1, 2); $dx^3 = hd\zeta$; $v_{x^j} = Vu_j$; $v_{x^3} = Vw$; $p = (\rho\nu r_0V/h^2)\cdot\Pi$, pressure; τ_{ij}, viscous stress tensor components; $Re_* = Vh/\nu$; $Re = \varepsilon Re_*$; S, median surface region in which liquid motion is studied; Γ, boundary of region S; $2q_j$, components of liquid specific flux vector in gap; $q_j^{(k)}$, coefficients of asymptotic expansion of function q_j; $\Phi^{(k)}$, vector potential of $q_j^{(k)}$; $\bar{e}_j$, unit vector tangent to coordinate line ξ^j; $\bar{n}$, external unit normal vector to contour Γ; q_n, component of specific liquid flux vector along external normal; $(U^{(k)})^j$, $(U^{(k)})_j$, contravariant and covariant components of vector $u_j^{(k)} = H_j(U^{(k)})^j = H_j^{-1}(U^{(k)})_j$; $(U^{(k)})_{j,\ell}$, covariant derivative of covariant vector; ρ, liquid density; ν, kinematic viscosity coefficient; V, flow velocity scale.

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LOCAL HEAT EXCHANGE OF A CYLINDER IN A SLIGHTLY DUSTY FLOW

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A study is performed of thermal resistances of convective heat exchange and thermal conductivity caused by reduction in the intensity of heat exchange when loose deposits are formed on the cylinder.

In energy and heat utilization apparatus, the dynamic equilibrium of the layer of loose deposits formed on tube surfaces is determined primarily by precipitation from the flow of fine particles, accompanied by destruction of the layer by collisions of coarser ash particles. In a number of devices (for example, gas-cooled reactors with spherical shells) the flow becomes contaminated by micron-size particles of a narrow fractional composition due to wearing and ablation of surfaces drafted by the flow. In view of the complete absence of binding components in the flow deposits of increased friability and an anomalously loose structure are formed. The absence of reliable information on the structure of such loose deposits, their distribution over surfaces, and their effect on heat transport prohibits determination of the local heat-exchange mechanism and sufficiently precise explanation of reductions in heat-exchange intensity.

The present author and Spokoinyi [1] studied mean heat exchange of a cylinder with a cooled slightly dusty ($\mu < 2\cdot10^{-3}$ kg·sec/(kg·sec)) air–graphite flow under conditions where a loose friable deposit was formed. The dispersed material used was type S-1 natural graphite powder ($\bar{d}_s = 7$ μm, $d_{t\ min} = 1.8$ μm, $d_{t\ max} = 15$ μm; $\rho_s = 2000$ kg/m³). The polystyrene cylindrical tube ($D = 21\times1.5$ mm; $\lambda_w = 0.1165$ W/(m·K)) was located horizontally in a cooled descending flow. Heat removal from the cylinder wall was accomplished by pumping a coolant liquid.

The experiments on local heat exchange studied the effective heat-liberation coefficient

$$\alpha^*_{fi} = \frac{2\lambda_w\,(t^e_{w\,i} - t^{in}_{w\,i})}{D_e\,(\bar{t}_f - t^e_{w\,i})\ln\dfrac{D_e}{D_{in}}} \tag{1}$$

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